1.

$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants *A*, *B* and *C*.

(4)

(b) (i) Hence find $\int f(x)\,dx$.

(3)

(ii) Find $\int_0^2 f(x)\,dx$ in the form 1n *k*, where *k* is a constant.

(3)
(Total 10 marks)

2.

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$$

(a) Find the values of the constants *A*, *B* and *C*.

(4)

(b) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx$ is 2 + ln *k*, giving the value of the constant *k*.

(6)
(Total 10 marks)

3.

$$f(x) = \frac{9+4x^2}{9-4x^2}, \qquad x \neq \pm\tfrac{3}{2}.$$

(a) Find the values of the constants *A*, *B* and *C* such that

$$f(x) = A + \frac{B}{3+2x} + \frac{C}{3-2x}, \qquad x \neq \pm\tfrac{3}{2}.$$

(4)

(b) Hence find the exact value of

$$\int_{-1}^{1} \frac{9+4x^2}{9-4x^2}\,\mathrm{d}x$$

(5)
(Total 9 marks)

4. (a) Express $\dfrac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

(b) Hence find the exact value of $\displaystyle\int_{2}^{6} \frac{5x+3}{(2x-3)(x+2)}\,\mathrm{d}x$, giving your answer as a single logarithm.

(5)
(Total 8 marks)

¢ 0**5.**

$$\mathrm{f}(x) = \frac{25}{(3+2x)^2(1-x)}, \quad |x| < 1.$$

(a) Express f(x) as a sum of partial fractions.

(4)

(b) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$.

(5)

(c) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^2. Give each coefficient as a simplified fraction.

(7)
(Total 16 marks)

6.

$$\mathrm{g}(x) = \frac{5x+8}{(1+4x)(2-x)}.$$

(a) Express g(x) in the form $\dfrac{A}{(1+4x)} + \dfrac{B}{(2-x)}$, where A and B are constants to be found.

(3)

The finite region R is bounded by the curve with equation $y = \text{g}(x)$, the coordinate axes and the line $x = \frac{1}{2}$.

(b) Find the area of R, giving your answer in the form $a \ln 2 + b \ln 3$.

(7)
(Total 10 marks)

7. $\text{f}(x) = \dfrac{1+14x}{(1-x)(1+2x)}, \qquad |x| < \dfrac{1}{2}.$

(a) Express $\text{f}(x)$ in partial fractions.

(3)

(b) Hence find the exact value of $\int_{\frac{1}{6}}^{\frac{1}{3}} \text{f}(x)\, \text{d}x$, giving your answer in the form $\ln p$, where p is rational.

(5)

(c) Use the binomial theorem to expand $\text{f}(x)$ in ascending powers of x, up to and including the term in x^3, simplifying each term.

(5)
(Total 13 marks)

1. (a) f(x) = $\frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$

$4 - 2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$ M1

A method for evaluating one constant M1

$x \to -\frac{1}{2}$, $5 = A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A = 4$ any one correct constant A1

$x \to -1$, $6 = B(-1)(2) \Rightarrow B = -3$

$x \to -3$ $10 = C(-5)(-2) \Rightarrow C = 1$ all three constants correct A1 4

(b) (i) $\int \left(\frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3} \right) dx$

$= \frac{4}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + C$ A1 two

ln terms correct M1 A1ft

All three ln terms correct and "+C" ; ft constants A1ft 3

(ii) $\left[2\ln(2x+1) - 3\ln(x+1) + \ln(x+3)\right]_0^2$

= (2ln 5 – 3ln 3 + ln 5) – (2ln1 – 3ln1 + ln 3) M1

= 3ln5 – 4ln3

$= \ln\left(\frac{5^3}{3^4}\right)$ M1

$= \ln\left(\frac{125}{81}\right)$ A1 3

[10]

2. (a) **Way 1**

A method of long division gives,

$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv 2 + \frac{4}{(2x+1)(2x-1)}$

$\frac{4}{(2x+1)(2x-1)} \equiv \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$

$4 \equiv B(2x-1) + C(2x+1)$

or their remainder, $Dx + E \equiv B(2x-1) + C(2x+1)$

Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$

Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$ 4

B1 $A = 2$

M1 Forming any one of these two identities. Can be implied.

See note below

A1 either one of $B = -2$ or $C = 2$

A1 both B and C correct

Aliter (a) Way 2

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$$

See below for the award of B1

$2(4x^2 + 1) \equiv A(2x + 1)(2x - 1) + B(2x - 1) + C(2x + 1)$

Equate x^2, $8 = 4A \Rightarrow A = 2$

Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$

Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$

B1 ***decide to award B1 here!! ...***
... for $A = 2$

M1 Forming this identity. Can be implied.
If a candidate states one of either B or C correctly then the method mark M1 can be implied.

See note below

A1 either one of $B = -2$ or $C = 2$

A1 both B and C correct

(b) $$\int \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \int 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}\,dx$$

$$= 2x - \frac{2}{2}\ln 2(2x+1) + \frac{2}{2}\ln(2x-1)(+c)$$ 6

M1* Either $p\ln(2x + 1)$ or $q\ln(2x - 1)$
or either $p \ln 2x + 1$ or $q \ln 2x - 1$
Some candidates may find rational values for B and C.
They may combine the denominator of their B or C with $(2x + 1)$ or $(2x - 1)$. Hence:

Either $\frac{a}{b(2x-1)} \to k\ln(b(2x-1))$ or $\frac{a}{b(2x+1)} \to k\ln(b(2x+1))$

is okay for M1.
Candidates are not allowed to fluke $-\ln(2x + 1) + \ln(2x - 1)$ for A1. Hence **cso**. If they do fluke this, however, they can gain the final A1 mark for this part of the question.

B1ft $A \to Ax$

A1cso&aef $-\frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)$ or $-\ln(2x+1) + \ln(2x-1)$

See note below.

$$\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = [2x - \ln(2x+1) + \ln(2x-1)]_1^2$$

$= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$

$= 2 + \ln 3 + \ln 3 - \ln 5$

$= 2 + \ln\left(\frac{3(3)}{5}\right)$

$= 2 + \ln\left(\frac{9}{5}\right)$

depM1* Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay).

M1 Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for their numerical expression.
To award this M1 mark, the candidate must use the appropriate law(s) of logarithms for their ln terms to give a **one single** logarithmic term. Any error in ***applying the laws of logarithms*** would then earn M0.
Note: This is not a dependent method mark.

A1 $2 + \ln\left(\frac{9}{5}\right)$ or $2 - \ln\left(\frac{5}{9}\right)$ **and** k stated as $\frac{9}{5}$.

[10]

3. (a) $\frac{9+4x^2}{9-4x^2} = -1 + \frac{18}{(3+2x)(3-2x)}$, so $A = -1$ B1

Uses $18 = B(3 - 2x) + C(3 + 2x)$ and attempts to find B and C M1

$B = 3$ and $C = 3$ A1 A1 4

Or

Uses $9 + 4x^2 = A(9 - 4x^2) + B(3 - 2x) + C(3 + 2x)$ and attempts to find A, B and C M1
$A = -1$, $B = 3$ and $C = 3$ A1, A1, A1 4

(b) Obtains $Ax + \frac{B}{2}\ln(3 + 2x) - \frac{C}{2}\ln(3 - 2x)$ M1 A1

Substitutes limits and subtracts to give $2A + \frac{B}{2}\ln(5) - \frac{C}{2}\ln(\frac{1}{5})$ M1 A1ft

$= -2 + 3\ln5$ or $-2 + \ln125$ A1 5

[9]

4. (a) $\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$
$5x + 3 = A(x + 2) + B(2x - 3)$
Substituting $x = -2$ or $x = \frac{3}{2}$ and obtaining A or B; or equating coefficients and solving a pair of simultaneous equations to obtain A or B. M1
$A = 3$, $B = 1$ A1, A1 3
If the cover-up rule is used, give M1 A1 for the first of A or B found, A1 for the second.

(b) $\int \frac{5x+3}{(2x-3)(x+2)} \mathrm{d}x = \frac{3}{2}\ln(2x-3) + \ln(x+2)$ M1 A1ft

$[...]_2^6 = \frac{3}{2}\ln 9 + \ln 2$ M1 A1

$= \ln 54$ cao A1 5

[8]

5. (a) Method using either M1

$\frac{A}{(1-x)} + \frac{B}{(2x+3)} + \frac{C}{(2x+3)^2}$ **or** $\frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2}$

A = 1 B1,

C = 10 , B = 2 **or** D = 4 and E = 16 A1, A1 4

(b) $\int [\frac{1}{1-x} + \frac{2}{2x+3} + 10(2x+3)^{-2}]dx$ **or** $\int \frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2}dx$ M1

$-\ln|1-x| + \ln|2x+3| - 5(2x+3)^{-1}$ (+c) **or**

$-\ln|1-x| + \ln|2x+3| - (2x+8)(2x+3)^{-1}$ (+c) M1 A1 ftA1 ftA1 ft 5

(c) Either

$(1-x)^{-1} + 2(3+2x)^{-1} + 10(3+2x)^{-2} =$ M1

$1 + x + x^2 + \ldots$ A1 ft

$+\frac{2}{3}(1 - \frac{2x}{3} + \frac{4x^2}{9}...)$ M1 A1 ft

$+\frac{10}{9}(1 + (-2)(\frac{2x}{3}) + \frac{(-2)(-3)}{2}(\frac{2x}{3})^2 + ...)$ A1 ft

$= \frac{25}{9} - \frac{25}{27}x + \frac{25}{9}x^2...$ M1 A1 7

Or

$25[(9+12x+4x^2)(1-x)]^{-1} = 25[(9+3x-8x^2-4x^3)]^{-1}$ M1 A1

$\frac{25}{9}\left[1 + \frac{3x}{9} - \frac{8x^2}{9} - \frac{4x^3}{9}\right]^{-1} = \frac{25}{9}\left[1 - \left(\frac{3x}{9} - \frac{8x^2}{9} - \frac{4x^3}{9}\right) + \left(\frac{x^2}{9}..\right).\right]$ M1 A1 A1

$= \frac{25}{9} - \frac{25}{27}x + \frac{25}{9}x^2$ M1, A1 7

[16]

6. (a) $A(2-x) + B(1+4x) = 5x + 8$ M1

$x = 2$ $9B = 18$ $\Rightarrow B = 2$ A1

$x = -\frac{1}{4}$ $\frac{9A}{4} = \frac{27}{4}$ $\Rightarrow A = 3$ A1 3

(b) Area = $\int_0^{\frac{1}{2}} g(x)\, dx$ + (attempt to integrate)

$= 3\int \frac{dx}{(1+4x)} + 2\int \frac{dx}{(2-x)}$

$= \frac{3}{4}\left[\ln(1+4x)\right]_0^{\frac{1}{2}} - 2\left[\ln(2-x)\right]_0^{\frac{1}{2}}$ A1 A1

$= \frac{3}{4}\ln 3 - 2\ln\left(\frac{3}{2}\right) + 2\ln 2$ M1 A1 A1

$= \frac{3}{4}\ln 3 - 2\ln 3 + 2\ln 2 + 2\ln 2$

$= 4\ln 2 - \frac{5}{4}\ln 3$ A1 7

[10]

7. (a) $\frac{1+14x}{(1-x)(1+2x)} \equiv \frac{A}{1-x} + \frac{B}{1+2x}$ and attempt A and or B M1

$A = 5$, $B = -4$ A1, A1 3

(b) $\int \frac{5}{1-x} - \frac{4}{1+2x}\, dx = [-5\ln|1-x| - 2\ln|1+2x|]$ M1 A1

$= \left(-5\ln\frac{2}{3} - 2\ln\frac{5}{3}\right) - \left(-5\ln\frac{5}{6} - 2\ln\frac{4}{3}\right)$ M1

$= 5\ln\frac{5}{4} + 2\ln\frac{4}{5}$

$= 3\ln\frac{5}{4} = \ln\frac{125}{64}$ M1 A1 5

(c) $5(1-x)^{-1} - 4(1+2x)^{-1}$ B1 ft

$= 5(1 + x + x^2 + x^3) - 4$

$\left(1 - 2x + \frac{(-1)(-2)(2x)^2}{2} + \frac{(-1)(-2)(-3)(2x)^3}{6} + \ldots\right)$ M1 A1

$= 1 + 13x - 11x^2 + 37x^3 \ldots$ M1 A1 5

[13]

1. Part (a) was well done with the majority choosing to substitute values of x into an appropriate identity and obtaining the values of *A*, *B* and *C* correctly. The only error commonly seen was failing to solve $5 = \frac{5}{4}A$ for A correctly. Those who formed simultaneous equations in three unknowns tended to be less successful. Any incorrect constants obtained in part (a) were followed through for full marks in part (b)(i). Most candidates obtained logs in part (b)(i). The commonest error was, predictably, giving $\int \frac{4}{2x+1}\,dx = 4\ln(2x+1)$, although this error was seen less frequently than in some previous examinations. In indefinite integrals, candidates are expected to give a constant of integration but its omission is not penalised repeatedly throughout the paper. In part (b)(ii) most applied the limits correctly although a minority just ignored the lower limit 0. The application of log rules in simplifying the answer was less successful. Many otherwise completely correct solutions gave 3 ln 3 as ln 9 and some "simplified" 3 ln 5 – 4 ln 3 to $\frac{3}{4}\ln\left(\frac{5}{3}\right)$.

2. This question was well done with many candidates scoring at least eight of the ten marks available.

In part (a), the most popular and successful method was for candidates to multiply both sides of the given identity by $(2x + 1)(2x – 1)$ to form a new identity and proceed with "Way 2" as detailed in the mark scheme. A significant proportion of candidates proceeded by using a method ("Way 1") of long division to find the constant *A*. Common errors with this way included algebraic and arithmetic errors in applying long division leading to incorrect remainders; using the quotient instead of the remainder in order to form an identity to find the constants *B* and *C*; and using incorrect identities such as $2(4x^2 + 1) \equiv B(2x – 1) + C(2x + 1)$.

In part (b), the majority of candidates were able to integrate their expression to give an expression of the form $Ax + p\ln(2x + 1) + q\ln(2x – 1)$. Some candidates, however, incorrectly integrated $\frac{B}{(2x+1)}$ and $\frac{C}{(2x-1)}$ to give either $B\ln(2x + 1)$ and $C\ln(2x – 1)$ or $2B\ln(2x + 1)$ and $2C\ln(2x – 1)$. A majority of candidates were able to substitute their limits and use the laws of logarithms to find the given answer. Common errors at this point included either candidates writing $–\ln(2x + 1) + \ln(2x – 1)$ as $\ln(4x^2 – 1)$; or candidates writing –ln 5 + ln 3 as either ±ln 15 or ±ln 8.

3. Many correct answers were seen to part (a). Candidates who used long division were generally less successful often leading to the misuse of $1+\frac{8x^2}{9-4x^2}$ to attempt partial fractions. The few candidates who ignored the 'hence' in part (b) made no progress in their integration, but would not have gained any marks anyway. Although candidates knew $\int \frac{1}{ax+b}\,dx = k\ln(ax+b)$, they were less accurate with the value of *k*. This question provided another challenge for candidates who were careless in their use of brackets whose answers often led to giving –[–(–1)] as +1. A few candidates ignored the requirement for an exact value.

4. Almost all candidates knew how to do this question and it was rare to see an incorrect solution to part (a) and nearly all could make a substantial attempt at part (b).

However the error $\int \frac{3}{2x-3}\,\mathrm{d}x = 3\ln(2x-3)$ was common. The standard of logarithmic work seen in simplifying the final answer was good and this, and the manipulation of exponentials elsewhere in the paper, is another area in which the standard of work has improved in recent examinations.

5. Most candidates chose an appropriate form for the partial fractions, and they demonstrated knowledge of how to find values for their constants. Where things did go wrong, it was often because of errors in arithmetic, but also in forming the correct numerator for the combined partial fractions – an extra factor of $2x + 3$ was popular.

Those who chose constant numerators for the fractions had the easier time in part (b), although there were frequent errors involving wrong signs and wrong constants in the integrals. Many candidates could not integrate $(3 + 2x)^{-2}$, and erroneously used a log. Few candidates with the two-fraction option could see how to make progress with the integration of the fraction with the quadratic denominator, though there were a number of possible methods.

In part (c) very few candidates achieved full marks for this part as many made errors with signs when expanding using the binomial. The most common mistake however was to take out the 3 from the brackets incorrectly. Not as $\frac{1}{3}$ and $\frac{1}{9}$, but as 3 and 9. However many candidates did manage to get some marks, for showing they were trying to use negative powers and also for the $1 + x + x^2$, and for trying to collect all their terms together at the end.

6. No Report available for this question.

7. No Report available for this question.